Determine piston cubic inch displacement
Program Task: Diagnose engine drivability problems.

## Program Associated Vocabulary:

 STROKE, VOLUME, BORE, DISPLACEMENT
## Program Formulas and Procedures:

cu.in. displacement of a cylinder $=$

$$
\mathrm{CID}=\frac{\text { bore }^{2} \pi \text { stroke }}{4} \text { or } \mathrm{V}=\pi \mathrm{r}^{2} \mathrm{~h}
$$

Piston bore (the diameter of the cylinder) squared times 3.14 (or $\pi$ ) times the stroke (the distance the piston travels up and down) divided by 4 . We divide by 4 because we squared the diameter.


$$
\begin{aligned}
& \text { Bore }=5, \text { Stroke }=10 \\
& \text { ci. } i n .=\frac{\text { bore }^{2} \pi \text { stroke }}{4} \\
& \text { ci. } i n . ~=\frac{5^{2} \pi 10}{4} \\
& \text { ci. } i n . ~=25 \times \pi \times 10 \div 4 \\
& \text { ci. } i n . ~=196.3495 \text { in. }^{3}\left(196.4 \text { in. }^{3} \text { rounded }\right)
\end{aligned}
$$

## Explain volume formulas and use them to solve problems

## PA Core Standard: CC.2.3.HS.A. 12

Description: Explain volume formulas and use them to solve problems.

## Math Associated Vocabulary:

AREA, VOLUME, LENGTH, WIDTH, HEIGHT, RECTANGULAR, ROUND, CYLINDRICAL, BASE, RADIUS, RECTANGULAR PRISM

## Formulas and Procedures:

Volume:
Cylinder:
$\mathrm{V}=\pi \mathrm{r}^{2} \mathrm{~h}$


Cone:
$\mathrm{V}={ }^{1} / 3 \pi \mathrm{r}^{2} \mathrm{~h}$


Rectangular Prism:
$\mathrm{V}=\mathrm{l} \mathrm{wh}$


Sphere:
$\mathrm{V}={ }^{4} / 3 \pi \mathrm{r}^{3}$


Pyramid:
$\mathrm{V}=1 / 3$ (area of the base)h
$\mathrm{h}=$ height $\quad \mathrm{b}=$ base
$\ell=$ slant length or slant height


## Example:

How many cubic inches of air can a beach ball hold if it has a diameter of 14 inches? Round to the nearest whole number.

## Steps to finding volume:

1. Identify the solid. (sphere)
2. Write the formula for calculating the volume of that solid using the formula sheet.
$\mathrm{V}={ }^{4} / 3 \pi \mathrm{r}^{3}$
3. Identify what information you are given in the example.

Given: diameter (d) $=14$ "
4. Solve for radius using the formula radius $(r)=1 / 2$ (diameter).
$r=1 / 2 \times 14=7$
5. Perform the necessary mathematical operations to obtain your answer.
$\mathrm{V}={ }^{4} / 3 \pi \mathrm{r}^{3}={ }^{4} / 3(3.14)\left(7^{3}\right)=1,436 \mathrm{in}^{3}$
6. Write the appropriate unit after your answer.

1,436 in $^{3}$

## Instructor's Script - Comparing and Contrasting

Whether calculating piston volume or mathematical volume, the math concepts and the formulas used are very similar. Occasionally, automotive texts describe volume formulas in terms of diameter (d) instead of radius (r). When this happens, $\boldsymbol{\pi}$ is often replaced with 0.7854, (which is the same as $\pi / 4$ ): Cylinder volume: $\mathrm{V}=\pi \mathrm{r}^{2} \mathrm{~h}=0.7854 \mathrm{dh}$

If the volume involves a circular or spherical shape (cylinder, sphere, cone), then $\pi$ will be part of the calculation. The best way to use $\pi$ in your calculations is to use a $\pi$ key on the calculator, if available. Otherwise, using 3.14 as an approximation is fine.
The mathematical formulas for volume indicate a certain type of orientation that may not match the application in question. For example, h will designate height of a cylinder, but if the cylinder is horizontal, h will be the same as the length:

Both cylinders have same volume:


Use this formula for question three on page three: $V=\frac{1}{3} \pi r^{2} h$


## Common Mistakes Made By Students:

Students may use an incorrect formula to solve a problem: To rectify these errors have the students correctly identify the type of object they are dealing with and use the appropriate formula. Frequently two formulas may be needed for complex problems.

Most volume formulas need radius (r), not diameter (d): If you are given a diameter, halve it to get the radius before using the formula. Example: Diameter is 10 inches; radius $=10 \div 2=5$ inches

Using consistent units: If the problem asks for the answer in square feet instead of square inches, be sure to either convert your given measurements into feet first (inches $\div 12=$ feet) or convert your cubic inch answer into cubic feet (sq. in. $\div 1,728=\mathrm{cu}$. ft.).

1 cubic foot is a box 12 inches by 12 inches by 12 inches, so the calculation to convert cubic inches to cubic feet (or vice versa) must use $12 \times 12 \times 12=1,728 \mathrm{cu}$. in. per cu. ft.
1 cubic yard is a box 3 feet by 3 feet by 3 feet, so the conversion of cubic feet to cubic yards uses 27 cu . ft. per cu. yd.

## CTE Teacher's Extended Discussion

Some of the automotive technology topics that require an understanding of volume as well as the ability to calculate volume in a variety of scenarios would include (but would definitely not be limited to):

1. Volume of cylinders/combustion chamber
2. Volume/displacement (propane, butane, oxygen, acetylene, nitrogen, diesel, etc., differences between gas/liquid uses).
3. Tubing and piping systems (AC suction and liquid lines)
4. Coolant systems (calculating anti-freeze temperature/pressure relationships within closed loop systems).
5. How many examples can you add?

| Problems Career and Tech | Solutions |
| :---: | :---: |
| 1. What is the cubic inch displacement of a single cylinder with a 3.5 " bore and a 4.5 " stroke; what is the total engine displacement if the engine has 8 cylinders? |  |
| 2. Your car's engine is a " 301. ." 301 means the engine displaces 301 in 3 . You find the bore $=4 "$, \& stroke $=3^{\prime \prime}$ What is the displacement of one cylinder? How many cylinders does this engine have? |  |
| 3. The eight orange traffic cones used on a test track to measure brake stopping distance need to be filled with concrete to keep them from moving. The height $(\mathrm{h})=36$ " \& the diameter (d) is 15 ". What is the volume of each pyramid in in 3 ? What is the volume of all 8 pyramids in in 3 \& in ft3? |  |
| Problems Related, Gener | Solutions |
| 4. One soup can has ad $=3$ inches and $\mathrm{h}=4$ inches; another soup can has a $\mathrm{d}=4$ inches and $\mathrm{h}=3$ inches. Which can holds more soup? |  |
| 5. A size 7 regulation basketball has a $\mathrm{d}=9.39$ inches. What is the volume of the basketball? |  |
| 6. How much water would you need to fill a rectangular fish tank with a height of 16.5 inches, a length of 32 inches, and a width of 8.5 inches? |  |
| Problems PA Core | Solutions |
| 7. Find the volume of a cylinder if $\mathrm{d}=12.5$ inches and $\mathrm{h}=28.45$ inches. |  |
| 8. Find the volume of a sphere if $\mathrm{d}=27.75$ inches. |  |
| 9. Find the volume of a 4 -sided pyramid with a square base side of 10 inches, and a height of 25 inches. |  |


| Problems Career and Tech | ical Math Concepts Solutions |
| :---: | :---: |
| 1. What is the cubic inch displacement of a single cylinder with a 3.5 " bore and a 4.5 " stroke; what is the total engine displacement if the engine has 8 cylinders? | $\begin{aligned} & \mathrm{V}=\pi \times 1.75^{2} \times 4.5=43.295 \mathrm{in} . .^{3} \text { or } \\ & \mathrm{CID}=\frac{3.5^{2} \square \pi \sqsubset 4.5}{4}=43.295 \mathrm{in} . .^{2} \\ & \text { cu.in. displacment }=43.3 \times 8=346 \mathrm{in} .{ }^{3} \end{aligned}$ |
| 2. Your car's engine is a " 301 ." 301 means the engine displaces $301 \mathrm{in}^{3}$. You find the bore $=4$ ", \& stroke $=3$ " What is the displacement of one cylinder? How many cylinders does this engine have? <br> This engine has $\qquad$ cylinders. | Piston Displacement (V) $=\pi \times 2^{2} \times 3=37.7$ in. ${ }^{3}$ or $\begin{aligned} & \mathrm{V}=\frac{4^{2} \times \pi \times 3}{4}=37.7 \mathrm{in.}^{3} \\ & 301 \div 37.7=7.98(\text { an " } 8 \text { " cylinder engine }) \end{aligned}$ |
| 3. The eight orange traffic cones used on a test track to measure brake stopping distance need to be filled with concrete to keep them from moving. The height $(\mathrm{h})=36^{\prime \prime}$ \& the diameter (d) is 15 ". What is the volume of each pyramid in in ${ }^{3}$ ? What is the volume of all 8 pyramids in in ${ }^{3} \&$ in $^{\mathrm{ft}^{3}}$ ? | $\begin{aligned} & \mathrm{V}=\frac{1}{3} \pi \mathrm{r}^{2} \mathrm{~h} \rightarrow \mathrm{~V}=\frac{1}{3} \pi \times 7.5^{2} \times 36 \rightarrow \mathrm{~V}=2121 \mathrm{in.} .^{3} \\ & 2121 \times 8=16968 \text { in. }{ }^{3} \text { (There are } 1728 \text { cu.in. in } 1 \text { cu.ft.[144×12]) } \\ & 16968 \div 1728=9.8 \text { rounded to } 10 \mathrm{ft.}^{3} \end{aligned}$ |
| Problems $\quad$ Related, Generic Math Concepts $\quad$ Solutions |  |
| 4. One soup can has a d $=3$ inches and $\mathrm{h}=4$ inches; another soup can has a $\mathrm{d}=4$ inches and $\mathrm{h}=3$ inches. Which can holds more soup? |  |
| 5. A size 7 regulation basketball has $\mathrm{ad}=9.39$ inches. What is the volume of the basketball? | $\begin{aligned} & \mathrm{V}=\frac{4}{3} \times \pi \times \mathrm{r}^{3} \longrightarrow \mathrm{~V}=1.333 \times \pi \times 4.695^{3} \\ & \mathrm{~V}=1.333 \times \pi \times 103.5 \quad \mathrm{~V}=433.43 \mathrm{in} .^{3} \end{aligned}$ |
| 6. How much water would you need to fill a rectangular fish tank with a height of 16.5 inches, a length of 32 inches, and a width of 8.5 inches? | $\mathrm{V}=\mathrm{lwh} \quad \mathrm{V}=(32)(8.5)(16.5)=4,488 \mathrm{in}^{3}$ |
| Problems PA Core Math Look Solutions |  |
| 7. Find the volume of a cylinder, $\mathrm{d}=12.75$ inches and $\mathrm{h}=28.45$ inches. | $\begin{aligned} & \mathrm{V}=\pi \mathrm{r}^{2} \mathrm{~h} \\ & \mathrm{~V}=\pi \times 6.375^{2} \times 28.45=3632.39 \mathrm{in.}^{3} \end{aligned}$ |
| 8. Find the volume of a sphere if $\mathrm{d}=27.75$ inches. | $\begin{array}{ll} \mathrm{V}=\frac{4}{3} \times \pi \times \mathrm{r}^{3} & \mathrm{~V}=1.333 \times \pi \times 13.875^{3} \\ \mathrm{~V}=1.333 \times \pi \times 2671.15 & \mathrm{~V}=11186 \mathrm{in} .^{3} \end{array}$ |
| 9. Find the volume of a 4 -sided pyramid with a square base side of 10 inches, and a height of 25 inches. | $\mathrm{V}=\frac{1}{3}(\text { area of base }) \mathrm{h} \rightarrow \mathrm{~V}=\frac{1}{3}(10)(10)(25)=833.33 \mathrm{in.}^{3}$ |

